(* Appendix 1.　H.Ira : The Development of the Two Circle Roller in a Numerical Way　*)

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(* APPENDIX 1 *)
(* Select all and copy of this paper,
and paste on the MATHEMATICA in TEXT type,
then MATHEMATICA program will be run. *)

(* The development of the Two-Circle-Roller with a numerical integration *)

Off[General::spell]
r = 1.0;
(* "r" is a radius of the Two-Circle-Roller, and any value is permitted,
    1.0 is an example.*)
j = 100;
(* "j" is a discrete rate of a numerical integration *)

(* the inclination angle θ[t] of a tangential line
   of the curve Cad based on (36) *)

a = Sqrt[2] + 1;
b = Sqrt[2] - 1;
k = -1;
ϕ[t_] := ArcSin[Sqrt[Sqrt[2] - 1]Tan[t/2]];
θ[t_] := -2*Sqrt[2]*
(EllipticF[φ[t], k] - EllipticPi[-a, φ[t], k] - EllipticPi[-b, φ[t], k])

(* Sd(π/2)is center of a point - reflection,
   and let coordinates be(Xc[π/2], Yc[π/2]) *)

Xc[π/2] = r*(NIntegrate[Cos[θ[τ]], {τ, 0, Pi/2}] +
Sqrt[1 + Sqrt[2]Cos[t]]*(Cos[θ[t]]cosβ[t] - Sin[θ[t]]sinβ[t]))/. t -> Pi/2 //
N;
Yc[π/2] = r*(NIntegrate[Sin[θ[τ]], {τ, 0, Pi/2}] +
Sqrt[1 + Sqrt[2]Cos[t]]*(Sin[θ[t]]cosβ[t] + Cos[θ[t]]sinβ[t]))/. t -> Pi/2 //
N;


(*  The list of the curve Cad with a numerical integration (37)and(38) *)
 listXa1 =
   Table[r* NIntegrate[Cos[θ[t]], {t, 0, i*(Pi/(2*j))}], {i, 0, j}];
 listYa1 = Table[r* NIntegrate[Sin[θ[t]], {t, 0, i*(Pi/(2*j))}], {i, 0, j}];
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(* The list of the curve Cbd with a numerical integration (39)a and(40)a *)

sinβ[t_] := Sqrt[((Sqrt[2] + Cos[t])^2 - 1)/(2 + 2Sqrt[2]Cos[t])]
cosβ[t_] := Sin[t]/Sqrt[2 + 2Sqrt[2]Cos[t]]
ab[t_] := r *Sqrt[2](Sqrt[2] + Cos[t])/Sqrt[1 + Sqrt[2]Cos[t]]
(* "ab[t]" is an absolute value of the vector QA *)

abcosθβ[t_] := ab[t]*(Cos[θ[t]]*cosβ[t] - Sin[θ[t]]*sinβ[t])
absinθβ[t_] := ab[t]*(Sin[θ[t]]*cosβ[t] + Cos[θ[t]]*sinβ[t])

listabcosθβ = Table[abcosθβ[t] /. t -> i*(Pi/(2*j)) // N, {i, 0, j}];
listabsinθβ = Table[absinθβ[t] /. t -> i*(Pi/(2*j)) // N, {i, 0, j}];

listXb1 = listXa1 + listabcosθβ;
listYb1 = listYa1 + listabsinθβ;

(* Cmad4, Cmad3, Cmad2, Cmad1, Cad1, Cad2, Cad3, Cad4
   are segments of curve Cad *)
listXa2 = Table[2*Xc[π/2] - listXb1[[i]], {i, 1, j + 1}];
listYa2 = Table[2*Yc[π/2] - listYb1[[i]], {i, 1, j + 1}];

listXa3 = Table[4*Xc[π/2] - listXa2[[i]], {i, 1, j + 1}];
listYa3 = listYa2;

listXa4 = Table[4*Xc[π/2] - listXa1[[i]], {i, 1, j + 1}];
listYa4 = listYa1;

listCad1 = Transpose[{listXa1, listYa1}];
listCad2 = Transpose[{listXa2, listYa2}] // Reverse;
listCad3 = Transpose[{listXa3, listYa3}];
listCad4 = Transpose[{listXa4, listYa4}] // Reverse;
listCad1m = Transpose[{-listXa1, listYa1}] // Reverse;
listCad2m = Transpose[{-listXa2, listYa2}];
listCad3m = Transpose[{-listXa3, listYa3}] // Reverse;
listCad4m = Transpose[{-listXa4, listYa4}];

listCad =
   Partition[
    Flatten[{listCad4m, listCad3m, listCad2m, listCad1m,
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              listCad1, listCad2, listCad3, listCad4}], 2];
(* Invisible plotting of the curve Cad *)

curveCad =
 ListPlot[listCad, PlotJoined -> True, AspectRatio -> Automatic,
   PlotStyle -> AbsoluteThickness[2], DisplayFunction -> Identity]


 (* Cmbd4, Cmbd3, Cmbd2, Cmbd1, Cbd1, CBbd2, Cbd3, Cbd4
   are segments of curve Cbd *)

listXb2 = Table[2*Xc[π/2] - listXa1[[i]], {i, 1, j + 1}];
listYb2 = Table[2*Yc[π/2] - listYa1[[i]], {i, 1, j + 1}];

listXb3 = Table[4*Xc[π/2] - listXb2[[i]], {i, 1, j + 1}];
listYb3 = listYb2;

listXb4 = Table[4*Xc[π/2] - listXb1[[i]], {i, 1, j + 1}];
listYb4 = listYb1;

listCbd1 = Transpose[{listXb1, listYb1}];
listCbd2 = Transpose[{listXb2, listYb2}] // Reverse;
listCbd3 = Transpose[{listXb3, listYb3}];
listCbd4 = Transpose[{listXb4, listYb4}] // Reverse;
listCbd1m = Transpose[{-listXb1, listYb1}] // Reverse;
listCbd2m = Transpose[{-listXb2, listYb2}];
listCbd3m = Transpose[{-listXb3, listYb3}] // Reverse;
listCbd4m = Transpose[{-listXb4, listYb4}];

listCbd =
   Partition[
     Flatten[{listCbd4m, listCbd3m, listCbd2m, listCbd1m,
              listCbd1, listCbd2, listCbd3, listCbd4}], 2];

(* invisible plotting of the curve Cbd *)

curveCbd =
   ListPlot[listCbd, PlotJoined -> True, AspectRatio -> Automatic,
     PlotStyle -> AbsoluteThickness[2], DisplayFunction -> Identity];
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(*  The list of the curve Sc based on (68) *)

sccosθβ[t_] :=
  r Sqrt[1 + Sqrt[2]Cos[t]]*(Cos[θ[t]]*cosβ[t] - Sin[θ[t]]*sinβ[t])
scsinθβ[t_] :=
  r Sqrt[1 + Sqrt[2]Cos[t]]*(Sin[θ[t]]*cosβ[t] + Cos[θ[t]]*sinβ[t])
listsccosθβ = Table[sccosθβ[t] /. t -> i*(Pi/(2*j)) // N, {i, 0, j}];
listscsinθβ = Table[scsinθβ[t] /. t -> i*(Pi/(2*j)) // N, {i, 0, j}];

(* Smc4, Smc3, Smc2, Smc1, Sc1, Sc2, Sc3, Sc4
   are segments of curve Sc *)

listXc1 = listXa1 + listsccosθβ;
listYc1 = listYa1 + listscsinθβ;

listXc2 = Table[2*Xc[π/2] - listXc1[[i]], {i, 1, j + 1}];
listYc2 = Table[2*Yc[π/2] - listYc1[[i]], {i, 1, j + 1}];

listXc3 = Table[4*Xc[π/2] - listXc2[[i]], {i, 1, j + 1}];
listYc3 = listYc2;

listXc4 = Table[4*Xc[π/2] - listXc1[[i]], {i, 1, j + 1}];
listYc4 = listYc1;

listSc1 = Transpose[{listXc1, listYc1}];
listSc2 = Transpose[{listXc2, listYc2}] // Reverse;
listSc3 = Transpose[{listXc3, listYc3}];
listSc4 = Transpose[{listXc4, listYc4}] // Reverse;
listSc1m = Transpose[{-listXc1, listYc1}] // Reverse;
listSc2m = Transpose[{-listXc2, listYc2}];
listSc3m = Transpose[{-listXc3, listYc3}] // Reverse;
listSc4m = Transpose[{-listXc4, listYc4}];

listSc = Partition[
     Flatten[{listSc4m, listSc3m, listSc2m, listSc1m,
             listSc1, listSc2, listSc3, listSc4}], 2];
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(* Invisible plotting of the curve Sc. *)
curveSc =
  ListPlot[listSc, PlotJoined -> True, AspectRatio -> Automatic,
    PlotStyle -> {Hue[0.66], AbsoluteThickness[2]},
    DisplayFunction -> Identity]

(* Left and right lines *)
pla = {-listXa4[[1]], listYa4[[1]]};
plb = {-listXb4[[1]], listYb4[[1]]};
pra = {listXa4[[1]], listYa4[[1]]};
prb = {listXb4[[1]], listYb4[[1]]};
leftline = Show[Graphics[Line[{pla, plb}]], DisplayFunction -> Identity]
rightline = Show[Graphics[Line[{pra, prb}]], DisplayFunction -> Identity]

(* A plotting of the Cad, Cbd, and Sc.  *)

Show[curveCad, curveCbd, curveSc, leftline, rightline,
  DisplayFunction -> $DisplayFunction]
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